Section 3.5

Exponential and Logarithmic Models

Some of the most common types of mathematical models involving exponential functions and logarithmic functions are as follows.

a) Exponential growth/decay model: $y = ae^{bx}$, $b > 0$ (growth model), $b < 0$ (decay model).

b) Gaussian model: $y = ae^{-(x-b)^2/c}$.

c) Logistic growth model: $y = \frac{a}{1+be^{-rx}}$.

d) Logarithmic models: $y = a + b\ln x$, $y = a + \log x$.

Problem 1. The value V (in millions of dollars) of a famous painting can be modeled by $V = 10e^{kt}$, where t represents the year, with $t = 0$ corresponding to 1990. In 2004, the same painting was sold for \$65 million. Find the value of k, and use this result to predict the value of the painting in 2010.

Problem 2. The number N of bacteria in a culture is modeled by $N = 200e^{kt}$ where t is the time in hours. If $N = 250$ when $t = 8$, estimate the time required for the population to double in size.

Problem 3. A Gateway FX6831 computer that costs \$1170 new has a book value of \$400 after 2 years.

a) Find the linear model $V = mt + b$

b) Find the exponential model $V = ae^{kt}$

Problem 4. The half-life of bismuth-210 is about 5 days. You begin with 20 grams of bismuth-210. Determine the number of days needed for your sample to decay to 4 grams.

Problem 5. The quantity, Q, of a substance decays according to the formula $Q = Q_0 e^{-kt}$, where t is in hours. The half-life of the substance is 8 hours. What is the value of k?

Problem 6. The learning curve for the number N of units produced per day after a new employee has worked t days is modeled by $N = 32(1 - e^{kt})$. After 18 days on the job, a new employee produces 22 units.

a) Find the learning curve for this employee (find the value of k).

b) How many days should pass before this employee is producing 28 units per day?

Problem 7. After discontinuing all advertising for an MP3 player in 2005, the manufacturer noted that sales began to drop according to the model $S = \frac{400{,}000}{1+0.5e^{kt}}$ where S represents the number of units sold and $t = 0$ represents 2005. In 2007, the company sold $250{,}000$ units.

a) Complete the model by solving for k.

b) Estimate the sales in 2012.

Homework: Read section 3.5, do #7, 15, 21, 23, 27, 31, 33, 39, 43, and do 58 and 59 if you ever plan to get a mortgage.